LITERATURE CITED

1. N. Kh. Zinnatullin and I. V. Flegentov, in: Abstracts of the Kirov KKhTI, No. 48, 140-144 (1972).
2. P. Rouch, Computational Hydrodynamics [Russian translation], Moscow (1980).
3. J. Happel and G. Brenner, Hydrodynamics with Small Reynolds Numbers [Russian translation], Moscow (1976).
4. L. G. Loitsyanskii, The Mechanics of Liquids and Gases [in Russian], Moscow (1973).
5. D. G. Pazhi and V. S. Galustov, The Fundamentals of Fluid Spray Engineering [in Russian], Moscow (1984).
6. Yu. I. Khavkin, Centrifugal Nozzles [in Russian], Leningrad (1976).

MOTION OF A RAREFIED GAS IN A PLANE CHANNEL IN THE PRESENCE
OF CONDENSATION ON THE CHANNEL WALLS

F. M. Sharipov and T. V. Shchepetkina
UDC 533.6.011.8

We examine the flow of a rarefied gas through a broad range of Knudsen numbers under the action of small pressure and temperature differences in a plane short channel, with provision made for the processes of evaporation and condensation at the channel walls.

The processing of mass transfer, in which provision is made for evaporation and condensation on the walls, have been studied in numerous papers, such as, for example [1-5]. The transport of gas between plane infinite plates is the subject of [1], while [2, 3] deal with the motion of a gas in an infinite pore and a number of simplifying assumptions have been made here; in [4] we find a study of the flow in a finite channel, but the gas flow rate and its dependence on the length of the channel and the flow regime have not been dealt with, and in this particular case the boundary conditions are specified for the ends of the channel. In [5] we find a study of the kinetics involved in the mass transfer that occurs under the action of a small pressure difference in a plane finite pore, with consideration given to vaporization and condensation at the walls and at the bottom of the pore. The boundary conditions are specified directly at the inlet to the pore. In the present paper we investigate the heat and mass transfer that arises under the action of small pressure and temperature drops across a finite channel, with consideration given to the evaporation and condensation that occurs on the channel walls over a broad range of Knudsen numbers. Unlike the earlier-cited studies, the flow of gas is treated here not only within the channel, but also in the regions externally adjacent to the channel.

Let us take a look at a plane channel of length ℓ, of height a, and infinite in width, as shown in Fig. 1, connecting two vessels containing the identical gas. At a rather large distance from the channel, the gas within the vessels is maintained under equilibrium conditions at pressures P_1 and P_2 and temperatures T_1 and T_2, respectively. Here the distribution functions are in the form of absolute Maxwellians:

$$f_i = \frac{P_i}{kT_i} \left(\frac{m}{2\pi kT_i} \right)^{3/2} \exp\left(-\frac{mv^2}{2kT_i} \right), \; i = 1, 2.$$

The walls of each of the vessels exhibit temperatures of T_1 and T_2, respectively. It is assumed that all of the molecules reaching the walls of the vessels are diffusely reflected, and absorbed as they impinge on the channel walls. The walls, in this case, radiate the molecules with the following distribution function:

$$f_w = f_1 \left[1 + \frac{x}{l} \frac{P_2 - P_1}{P_1} + \frac{x}{l} \left(\frac{mv^2}{2kT_1} - \frac{5}{2} \right) \frac{T_2 - T_1}{T_1} \right].$$

Physically this means that the temperature of the channel walls and the pressure of the vaporizing gas, i.e., the pressure of the saturated vapor at the temperature of the wall, change linearly from T_1 to T_2 and from P_1 to P_2, respectively. Such a situation is encountered, for example, when the surface of the channel is coated with a thin film of a vaporizing liquid. It is assumed that the drops in pressure and temperature are small $|P_2 - P_1|/P_1 \ll 1$, $|T_2 - T_1|/T_1 \ll 1$. We have to find the flow field and the flows of mass and heat for arbitrary Knudsen numbers.

In order to achieve a rigorous solution for the formulated problem, it is necessary to use the Boltzmann equation, but with arbitrary rarefaction of the gas the complex structure of the collision integral makes this impossible. Therefore, the collision integral is replaced with one that is simpler, representative, but retaining the fundamental properties of the Boltzmann equation. In this particular study we have used the so-called third-order S model [6], which provides for adequate description of the processes of heat and mass transfer simultaneously:

$$\mathbf{v} \frac{\partial f}{\partial \mathbf{r}} = \frac{P}{\eta} (f^+ - f),$$

$$f^+ = f^0 \left[1 + \frac{4}{15} \mathbf{S} \cdot \mathbf{W} \left(W^2 - \frac{5}{2} \right) \right], \quad \mathbf{S} = \frac{1}{n} \int \mathbf{W} W^2 f d\mathbf{v},$$

$$f^0 = n \left(\frac{m}{2\pi kT} \right)^{3/2} \exp(-W^2), \quad \mathbf{W} = \sqrt{\frac{m}{2kT}} (\mathbf{v} - \mathbf{u}),$$

$$n = \int f d\mathbf{v}, \quad n\mathbf{u} = \int \mathbf{v} f d\mathbf{v}, \quad T = \frac{2}{3k} \int \frac{mv^2}{2} f d\mathbf{v},$$

$$P = nkT, \quad \eta = \frac{1}{2} nm v_t \lambda.$$

Let us introduce dimensionless quantities: $\mathbf{c} = (m/2kT_1)^{1/2} \mathbf{v}$, $\mathbf{u}' = (m/2kT_1)^{1/2} \mathbf{u}$ for the velocities of the molecules and the gas, respectively, $\mathbf{q}' = [1/2n_1 m(2kT_1/m)^{3/2}]^{-1} \mathbf{q} = \mathbf{S}$ for the heat flux density, $x' = x/a$, $y' = ya$ for the coordinates, and $J_k' = [(m/2kT_1)^{1/2}/kn_1 a] J_k$, $\Lambda_{k\ell}' = [(m/2kT_1)^{1/2}/kn_1 a] \Lambda_{k\ell}$ for the thermodynamic flows and kinetic coefficients. The primes for the dimensionless quantities will subsequently be dropped.

In view of the smallness of the drops in pressure and temperature, we can represent the unknown distribution function in the form

$$f(\mathbf{r}, \mathbf{c}) = f_1 [1 + h(\mathbf{r}, \mathbf{c})], \quad |h| \ll 1.$$

Having substituted this form of the distribution function into the kinetic equation, we obtain the following linearized S model:

$$\mathbf{c} \frac{\partial h}{\partial \mathbf{r}} = L_s h - \delta h,$$

$$L_s h = \delta \left[v + 2\mathbf{c} \cdot \mathbf{u} + \left(\frac{4}{15} \mathbf{q} \cdot \mathbf{c} + \tau \right) \left(c^2 - \frac{5}{2} \right) \right],$$

where $\delta = \sqrt{\pi} a/2\lambda$ is the rarefaction parameter, inversely proportional to the Knudsen number. The moments of the distribution function in this case can be written in the form

$$[v; \mathbf{u}; \tau; \mathbf{q}] = \int f_1 h \left[\frac{2}{3} c^2; \mathbf{c}; \frac{2}{3} c^2 - 1; \mathbf{c} \left(c^2 - \frac{5}{2} \right) \right] d\mathbf{c}.$$

Here $v = (P - P_1)/P_1$, $\tau = (T - T_1)/T_1$. Having integrated the linearized S model along the characteristic, we obtain the following expression for the perturbation function:

$$h = \int_0^{s_0} (L_s h) \exp \left(-\frac{\delta s}{c_p} \right) \frac{ds}{c_p} + h_{bo} \exp \left(-\frac{\delta s_0}{c_p} \right),$$
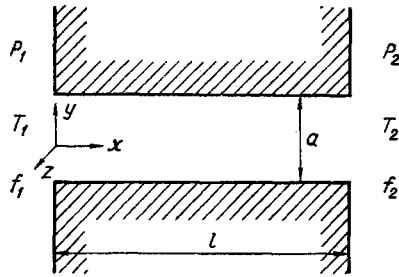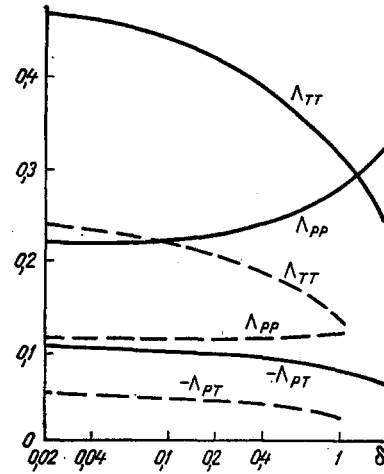
Fig. 1   Fig. 2

Fig. 1. The geometry of the problem.

Fig. 2. The kinetic coefficients Λ_{PP}, Λ_{PT}, and Λ_{TT} (dimensionless quantities) as functions of the reciprocal of the Knudsen number δ (a dimensionless quantity) for various channel lengths. The solid curves, L = 1; the dashed curves, L = 5.

where $c_p = c_x + c_y$; s is the distance from the point having the coordinates (x, y) in the direction $-c_p$; s_0 is the distance to the boundary of the flow field in this same direction; h_{b0} is the perturbation function set by the boundary conditions. Substitution of the last expression from the perturbation function in the determination of the moments of the distribution function gives us the following system of integral equations:

$$M_i(\mathbf{r}) = \frac{\delta}{\pi} \sum_{j=1}^{6} \int_0^{2\pi} \int_0^{s_0} K_{ij}(\mathbf{r},\ \mathbf{r}') M_j(\mathbf{r}')\,ds\,d\varphi + \frac{1}{\pi} \int_0^{2\pi} \Phi_i(\mathbf{r},\ \mathbf{r}_0')\,d\varphi, \qquad (1)$$

where $1 \leqslant i \leqslant 6$, $\mathbf{r}' = (x',\ y') = (x - s\cos\varphi,\ y - s\sin\varphi)$. We have introduced the following notation here:

$$M_1 = v(\mathbf{r}),\quad M_2 = u_x(\mathbf{r}),$$

$$M_3 = u_y(\mathbf{r}),\ M_4 = \tau(\mathbf{r}),\ M_5 = q_x(\mathbf{r}),\ M_6 = q_y(\mathbf{r}),$$

$$K_{11} = \frac{1}{3}(2I_2 + I_0),\quad K_{12} = \frac{2}{3}(2I_3 + I_1)\cos\varphi,$$

$$K_{14} = \frac{2}{3}I_4 - I_2 - \frac{1}{3}I_0,\quad K_{15} = \frac{4}{15}\left(\frac{2}{3}I_5 - I_3 - \frac{1}{3}I_1\right)\cos\varphi,$$

$$K_{21} = I_1\cos\varphi,\ K_{22} = 2I_2\cos^2\varphi,\ K_{24} = (I_3 - 2I_1)\cos\varphi,$$

$$K_{25} = \frac{4}{15}(I_4 - 2I_2)\cos^2\varphi,\ K_{41} = \frac{2}{3}(I_2 - I_0),$$

$$K_{42} = \frac{4}{3}(I_3 - I_1)\cos\varphi,\ K_{44} = \frac{2}{3}I_4 - 2I_2 + \frac{5}{3}I_0,$$

$$K_{45} = \frac{4}{15}\left(\frac{2}{3}I_5 - 2I_3 + \frac{5}{3}I_1\right)\cos\varphi,\ K_{51} = K_{24},\ K_{52} = \frac{15}{2}K_{25},$$

$$K_{54} = \left(I_5 - 4I_3 + \frac{9}{2}I_1\right)\cos\varphi,\ K_{55} = \frac{4}{15}\left(I_6 - 4I_4 + \frac{9}{2}I_2\right)\cos^2\varphi,$$

$$K_{ij} = K_{ij-1}\,\mathrm{tg}\,\varphi,\ K_{ji} = K_{j-1\,i}\,\mathrm{tg}\,\varphi,\ 1 \leqslant i \leqslant 6,\ j = 3;\ 6,$$

$$\Phi_i = \frac{1}{\cos\varphi}\left[\frac{1}{2}K_{i2}(\mathbf{r},\,\mathbf{r}_0')\,v_w + \frac{15}{4}K_{i5}(\mathbf{r},\,\mathbf{r}_0')\,\tau_w\right],\ 1\leqslant i\leqslant 6,$$

$$I_n = \int\limits_0^\infty c^n\exp\left(-c^2 - \frac{\delta s}{c}\right)dc,$$

$$s = (\mathbf{r}-\mathbf{r}')^{1/2},\quad \varphi = \mathrm{arctg}\,\frac{y-y'}{x-x'},$$

$$\mathbf{r}_0' = \begin{pmatrix} x_0' \\ y_0' \end{pmatrix} = \begin{pmatrix} x - s_0\cos\varphi \\ y - s_0\sin\varphi \end{pmatrix};$$

for $x_0' = 0$; L and $y_0' > 1/2$, $y_0' < -1/2$,

$$v_w = \frac{2}{\pi}\int\limits_{c_n<0}\exp(-c^2)\,h(c)\,dc,\quad \tau_w = \frac{T_2-T_1}{T_1},$$

for $0 \le x_0' \le L$ and $y_0' = \pm 1/2$,

$$v_w = \frac{x_0'}{L}\frac{P_2-P_1}{P_1},\quad \tau_w = \frac{x_0'}{L}\left(c^2 - \frac{5}{2}\right)\frac{T_2-T_1}{T_1}.$$

System (1) is solved numerically by the Krylov–Bogolyubov method [7]. The calculations have been performed on a computer with an error of no more than 2% for a channel length of L = 1 and 5 in a δ number range from 0.02 to 2. The calculation accuracy is monitored through comparison of the calculation results with various grids.

Let us analyze the problem under consideration from the standpoint of the thermodynamics of irreplaceable processes [8]. We will introduce two independent thermodynamic forces $X_P = \Delta P/P_1$, $X_T = \Delta T/T_1$. It has been demonstrated in [9] that in this case the thermodynamic flows are of the form

$$J_P = -\int\limits_{-1/2}^{1/2}u_x\,dy\bigg|_{x=L} - 2\int\limits_0^L u_y\frac{x}{L}\,dx\bigg|_{y=\frac{1}{2}},$$ (2)

$$J_T = -\int\limits_{-1/2}^{1/2}q_x\,dy\bigg|_{x=L} - 2\int\limits_0^L q_y\frac{x}{L}\,dx\bigg|_{y=\frac{1}{2}}.$$

We can see from relationships (2) that when the nonpenetration conditions are violated the thermodynamic flow Jp consists of two terms: the first term has the sense of a flow of mass through the inlet section of the channel. The second term is associated with the flow of mass to the channel walls, integrated with weight over the entire length, and dependent on the distribution of pressure in the vaporized gas. The thermodynamic flow J_T has an analogous form, and here, in the place of the gas velocity, we have the density of the heat flux, and replacing the pressure distribution of the vaporized gas, we have the wall temperature distribution. Only with these thermodynamic flows will the entropy yield in the system have the form $\sigma = J_P X_P + J_T X_T$, which corresponds to the conclusions from the thermodynamics of irreversible processes [8].

In view of the linearity of the problem, the moments of the distribution function can be expanded into the following component:

$$[v;\ u;\ \tau;\ q] = [v^P;\ u^P;\ \tau^P;\ q^P]\,X_P + [v^T;\ u^T;\ \tau^T;\ q^T]\,X_T.$$ (3)

Substituting (3) into (2) gives us a linear relationship for the flows Jp and J_T to the forces Xp and X_T:

$$J_P = \Lambda_{PP}\,X_P + \Lambda_{PT}\,X_T,$$
$$J_T = \Lambda_{TP}\,X_P + \Lambda_{TT}\,X_T.$$

The expressions for the kinetic coefficients Λ_{PP}, Λ_{PT}, Λ_{TP}, and Λ_{TT} will be analogous to the expressions for flows (2). Here the first subscript corresponds to the flow, while the second subscript corresponds to the component of the distribution-function moment substituted into the expressions. For nondiagonal kinetic coefficients the following reciprocity relationship is valid [9]:
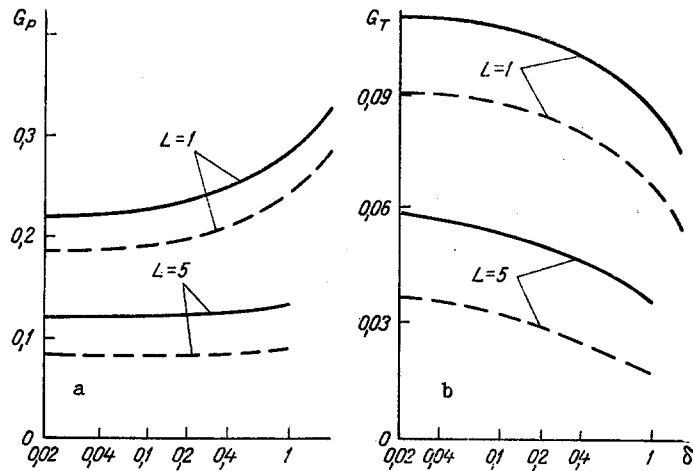
Fig. 3. The mass flow under the action of the pressure difference G_P (a dimensionless quantity) (a) and of the temperature difference G_T (a dimensionless quantity) (b) as a function of the reciprocal of the Knudsen number δ (a dimensionless quantity) at various sections of the channel. The solid curves represent the inlet section to the channel, while the dashed curves represent the central portion of the channel.

TABLE 1. The TPD Index as a Function of δ

δ	$L=1$	$L=5$
0	0,5	0,5
0,02	0,488	0,481
0,04	0,477	0,465
0,1	0,452	0,426
0,2	0,420	0,376
0,4	0,370	0,315
1	0,274	0,216
2	0,187	

$$\Lambda_{PT} = \Lambda_{TP}, \tag{4}$$

and this can be derived from the general concepts of the linear thermodynamics of irreversible processes [8]. It was shown in [9] that for this particular system relationship (4) is a consequence of the self-conjugacy of the linearized Boltzmann collision operator and the reciprocity of the scattering center of the gas molecules at the walls of the channels and the vessels. The boundary conditions which we have adopted provide for the satisfaction of the second condition. In the kinetic equation, in the place of the exact collision operator, in this particular paper we make use of the model operator $L_s h$. It is not difficult to prove that the model operator is also self-conjugate, so that its utilization in the place of the more exact operator must therefore ensure fulfillment of relationship (4). Analysis of the calculations shows that within the limits of calculation accuracy, in all cases the Onsager reciprocity relationship (4) is satisfied. This condition has not been imposed on the solution earlier, its fulfillment must be treated as an additional criterion of the accuracy of these derived solutions.

We will dwell briefly on the physical sense of each kinetic coefficient. The coefficient Λ_{PP} corresponds to the transport of mass under the action of the pressure drop, Λ_{TT} corresponds to the transfer of heat as a consequence of the temperature difference, and Λ_{PT} corresponds to the transfer of mass as a result of the temperature drop. The coefficient Λ_{TP} can be ascribed to the transfer of heat that is due to the pressure drop and, according to (4), is equal to Λ_{PT}. Figure 2 shows the kinetic coefficients as functions of the δ number in graphic representation. All of the coefficients depend significantly on the channel length. The flows of mass through the lateral cross section of the channel are of practical interest, and these can be determined in the following manner:

$$G_P(x) = \int_{-1/2}^{1/2} u_x^P(x, y)\, dy,$$

$$G_T(x) = \int_{-1/2}^{1/2} u_x^T(x, y)\, dy.$$

Figures 3 and 4 show the mass flows G_P and G_T as functions of the reciprocal Knudsen number δ for various channel lengths in the entry and central sections. We can see that with an increase in channel length there is an increase in that fraction of the gas absorbed at the walls of the channel. With an increase in the δ number the mass flow generated by the pressure drop G_P increases, while that generated by the temperature drop G_T diminishes.

In this study we have also calculated the pressure and temperature fields. We found that the temperatures of the wall and of the gas near the walls are different from each other. This phenomenon is referred to as a temperature discontinuity and with small δ makes itself apparent to a greater extent. We have also noted that the pressure of the gas near the wall differs from the pressure of the saturated vapor at the temperature of the wall, i.e., on a level with the temperature discontinuity we have also a pressure discontinuity, which disappears in the viscous regime.

Let us examine the phenomenon of the thermomolecular pressure difference. Let the vessels which connect the channel exhibit sufficiently large, but finite, volumes. If various temperatures T_1 and T_2 are maintained in the vessels, in addition to the flow of heat, a mass flow G_T arises within the system, and this is directed out of the "cold" vessel into the "hot" vessel. Since an absence of discharge is assumed within these vessels, this flow of mass leads to an increase in the pressure in the "hot" vessel and to a reduction in pressure in the "cold" vessel. In turn, this will result in a mass flow G_P in the opposite direction. If the system reaches steady state with the passage of time, the pressures P_1 and P_2 in the vessels will be related to the temperatures by the relationship

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^\gamma. \tag{5}$$

This effect is known as the thermomolecular pressure difference (TPD) [8]. In this case, the concept is generalized to the case in which the nonpenetration condition in the channel is violated. The TPD exponent γ is found from the condition that the total mass flow in each vessel is equal to zero:

$$\left(\frac{\Delta P}{P_1} \int_{-1/2}^{1/2} u_x^P\, dy + \frac{\Delta T}{T_1} \int_{-1/2}^{1/2} u_x^T\, dy\right)\Bigg|_{x=0;\,L} = 0. \tag{6}$$

Let us note that for a steady state it is sufficient to satisfy this relationship only for a single vessel, since in view of the symmetry of the flow field for the second vessel it will be satisfied automatically. In this case this condition may be violated for other sections of the channel. An interesting situation may arise in the absence of symmetry, when condition (6) for both of the vessels is simultaneously unfulfilled. In this case, the system never reaches the steady state, given that no provision is made for the required removal or source of gas molecules in one of the vessels, since otherwise the pressure in the system will constantly increase or diminish.

Considering the smallness of the differences in pressure and temperature, for the TPD exponent we obtain the following expression from (5) and (6):

$$\gamma = -\frac{\displaystyle\int_{-1/2}^{1/2} u_x^T\, dy\ \Big|_{x=0}}{\displaystyle\int_{-1/2}^{1/2} u_x^P\, dy\ \Big|_{x=0}}.$$

The dependence of γ on the δ number is shown in Table 1. In the free-molecular regime in a channel of any length $\gamma = 0.5$, and in the intermediate regime γ diminishes as the length of the channel increases.

NOTATION

a, channel height; ℓ, channel length; $L = \ell/a$, reduced channel length; $r = (x, y)$; P, pressure; T, temperature; m, molecule mass; $v_t = (8kT/\pi m)^{1/2}$, mean thermal velocity; f, distribution function; λ, mean free path; k, Boltzmann constant; η, dynamic viscosity; c and , reduced velocities of molecules and gas; q, heat-flux density; J_k, thermodynamic flows; $\Lambda_{k\ell}$, kinetic coefficient; h, perturbation function; X_k, thermodynamic forces. Subscripts: x, y, longitudinal and transverse components.

LITERATURE CITED

1. T. Soga, Phys. Fluids, 11, No. 25, 1978-1986 (1982).
2. N. V. Pavlyukevich, G. E. Gorelik, V. V. Levdanskii, et al., The Physical Kinetics and Processes of Transfer in Phase Conversions [in Russian], Minsk (1980).
3. F. M. Sharipov and T. V. Shchepetkina, The Molecular Physics of Nonequilibrium Systems [in Russian], Novosibirsk (1984), pp. 15-20.
4. P. Gajewski and A. Wisniewski, Bull. Acad. Pol. Sci. Ser. Sci. Tech., 26, No. 5, 513-520 (1978).
5. F. M. Sharipov, T. V. Shchepetkina, and A. M. Makarov, Inzh.-Fiz. Zh., 53, No. 1, 11-15 (1987).
6. E. M. Shakhov, A Method of Studying the Motions of a Rarefied Gas [in Russian], Moscow (1974).
7. L. V. Kantorovich and V. I. Krylov, Approximate Methods of Higher Analysis [in Russian], Moscow—Leningrad (1962).
8. S. DeGroot and P. Mazur, Nonequilibrium Thermodynamics [Russian translation], Moscow (1964).
9. F. M. Sharipov and V. D. Akin'shin, Inzh.-Fiz. Zh., 55, No. 2, 314 (1988); submitted to VINITI March 18, 1988, No. 2123.

STATIC PRESSURE MEASUREMENT ERRORS WHEN DRAINAGE ORIFICES ARE USED

E. U. Repik and V. K. Kuzenkov

We present the results from an experimental study of the influence exerted by the characteristic dimensions of a drainage orifice on the static-pressure measurement error.

Static pressure in a moving medium is usually measured by means of drainage orifices located on the streamlined surface. However, the presence of such orifices on the streamlined surface unavoidably leads to the perturbation of the flow in the boundary layer near that orifice and, consequently, to a deviation in the measured static pressure from the true value. The size of the perturbation zone near the drainage orifice depends on the diameter (d) of the orifice. It was demonstrated in [1] that these perturbations are propagated primarily through the thickness of the boundary layer and the thickness of the perturbation zone in this case varied from d/10 to d/40.

According to [2], the streamline adjacent to the streamlined surface, as it descends into this drainage pore, leads to the appearance of a field of centrifugal forces, as a consequence of which the pressure within the hole exceeds the actual pressure. The instability of the process involved in the formation of a system of vortices within the static-pressure hole also exerts its influence on the magnitude of the static pressure. In a number of cases, it is possible for the Pitot effect to set in at the edge of this hole downstream. When the stream is detached from the leading edge of the drainage orifice, the measured pressure (P_{meas}) will be smaller than the true pressure (P_{tru}). All of this may